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LAGRANGIAN FORMALISM OF SCHRODINGER AND RELATIVISTIC QUANTUM EQUATIONS FOR FRICTIONAL MEDIUM AND THE LASING BESIDE ABSORPTION CONDITIONS

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ABSTRACT

Lagrargian formalism was utilized in this work to obtain Schrödinger, Klein-Gordon and Dirac equation for frictional medium. Fortunately these equations are typical to that obtained by others. These equations were solved for lasing and absorption conditions. These conditions show that lasing and absorption take place due to the presence of frictional relaxation time term. This conforms to common sense and physical reality, since friction effect enables the medium to absorb energy from the incident beam. This absorbed energy can cause population inversion and lasing

Keywords: *friction, Schrodinger equation, Klein-Gordon equation, Dirac equation, lasing, absorption.*

I. INTRODUCTION

Today quantum mechanics can be applied to most fields of science. As technology advances, an increasing number of new electronic and opto-electronic devices will operate in ways which can only be understood using quantum mechanics. Quantum mechanics allows large objects to tunnel through a thin potential barrier if the constituents of the object are prepared in a special quantum mechanical state[1]. The laws of physics are also taken care of the properties of individual atoms as well as the bulk matter[2]. Any material consists of a large number of atoms. Each atom consists of electrons, protons and neutrons. Atom and subatomic particles are described by the lows of quantum mechanics. For slow moving particles one utilizes Schrodinger equation. Fast particles can be described by relativistic quantum mechanics, spin fewer particles are described by Klein-Gordon equation, while those having spin are described by Dirac equation[3,4]. It is believed that the basic physical building blocks forming the world we live in may be categorized into particles of matter. All known elementary constituents of matter and transmitters of force are quantized. For example, energy, momentum, and angular momentum take on discrete quantized values. The electron is an example of a transmitter of force. Neutrons, protons, and atoms are composite particles made up of elementary particles of matter and transmitters of force. These composite particles are also quantized[1]. Lagrangian methods have been extensively studied for the solution of constrained convex variational problems as well as control problems[5]. Also Lagrangian formalism is therefore independent of coordinate transformations. Also for Quantum systems a description based on energies is deeper physically and leads conceptually to quantum field theory. A fundamental result of mechanics is the Lagrange equations[6,7]. In particular, the Lagrangian formalism makes symmetries and their physical consequences explicit and thus is a convenient tool when constructing different kinds of theories based on symmetries observed (or speculated to exist) in nature[8] . Many attempts were made to account for the effect of friction but they are not based on Lagrangian formalism[2]. This work is devoted to construct relativistic quantum equation for resistive medium using Lagrange equation.

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II. CLASSICAL SCHRÖDINGER EQUATION OF FRICTIONAL MEDIUM:

Recently many attempts were made to modify Schrodinger equation for frictional media[3]. These needs suggesting a suitable Lagrangian function that describes this equation. To do this consider the Lagrangian

$$L = i\hbar\psi\dot{\psi}^* + \frac{\hbar^2}{2m}\nabla\psi.\nabla\psi^* + V\psi\psi^* - \frac{i\hbar}{\tau}\psi\psi^* \qquad (2.1)$$

Where the generalized coordinate is assumed to be

$$q = \psi^* \qquad (2.2)$$

Thus according to equations (2.1) and (2.2) one gets

$$\frac{\partial L}{\partial\psi^*} = V\psi - \frac{i\hbar}{\tau}\psi$$
$$\frac{\partial L}{\partial\dot{\psi}^*} = i\hbar\psi \qquad (2.3)$$

Where

$$\nabla\psi = \frac{\partial\psi}{\partial x}\hat{\imath} + \frac{\partial\psi}{\partial y}\hat{\jmath} + \frac{\partial\psi}{\partial z}\hat{k}$$
$$= \partial_x\psi\hat{\imath} + \partial_y\psi\hat{\jmath} + \partial_z\psi\hat{k}$$

And

$$\nabla\psi^* = \partial_x\psi^*\hat{\imath} + \partial_y\psi^*\hat{\jmath} + \partial_z\psi^*\hat{k}$$

Thus

$$\nabla\psi.\nabla\psi^* = \partial_x\psi\partial_x\psi^* + \partial_y\psi\partial_y\psi^* + \partial_z\psi\partial_z\psi^*$$

Using (2.4) and (2.1), one gets

$$\frac{\partial L}{\partial\partial_x\psi^*} = \frac{\hbar^2}{2m}\partial_x\psi \qquad (2.5)$$

But the Euler-Lagrange equation takes the form

$$\frac{\partial L}{\partial\psi^*} - \partial_t\left[\frac{\partial L}{\partial\dot{\psi}^*}\right] - \partial_x\left[\frac{\partial L}{\partial\partial_x\psi^*}\right] = 0 \qquad (2.6)$$

Inserting (2.3) and (2.9) in (2.6) yields

$$-\partial_t[i\hbar\psi] - \partial_x\left[\frac{\hbar^2}{2m}\partial_x\psi\right] + V\psi - \frac{i\hbar\psi}{\tau} = 0 \qquad (2.7)$$

Hence

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi - \frac{i\hbar\psi}{\tau} \qquad (2.8)$$

This is Schrödinger equation of frictional medium as proposed by some researchers[3].

III. KLEIN-GORDON EQUATION OF FRICTIONAL MEDIUM:

Similarly Klein-Gordon equation for frictional, resistive medium can be found by suggesting the Lagrangian to be

$$L = m_\circ^2c^4\psi\psi^* + \frac{\hbar^2}{\tau^2}\psi.\psi^* - \frac{2\hbar^2}{\tau}\psi\dot{\psi}^* - \hbar^2\dot{\psi}\dot{\psi}^* + \hbar^2c^2\nabla\psi.\nabla\psi^* \qquad (3.1)$$

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Therefore

$$\frac{\partial L}{\partial \psi^*} = m_\circ{}^2 c^4 \psi + \frac{\hbar^2}{\tau^2}\psi$$
$$\frac{\partial L}{\partial \dot{\psi}^*} = -\frac{2\hbar^2}{\tau}\psi - \hbar^2\dot{\psi}$$
$$\frac{\partial L}{\partial \partial_x \psi^*} = \hbar^2 c^2 \nabla\psi \qquad (3.2)$$

Using again Euler- Lagrange equation

$$\partial L/(\partial\psi^{\wedge}*) - \partial\_t\,[\partial L/(\partial\psi^{\wedge}*)] - \partial\_x\,[\partial L/(\partial\partial\_x\,\psi^{\wedge}*)] = 0 \qquad (3.3)$$

Together with equation (3.2) yields

$$m_\circ{}^2 c^4\psi + \frac{\hbar^2}{\tau^2}\psi - \frac{\partial}{\partial t}\left[-\frac{2\hbar^2}{\tau}\psi - \hbar^2\dot{\psi}\right] - \frac{\partial}{\partial x}[\hbar^2 c^2\nabla\psi] = 0$$
$$\therefore \quad m_\circ{}^2 c^4\psi + \frac{\hbar^2}{\tau^2}\psi + \frac{2\hbar^2}{\tau}\frac{\partial\psi}{\partial t} + \hbar^2\frac{\partial^2\psi}{\partial t^2} - \hbar^2 c^2\nabla^2\psi = 0$$
$$\therefore \quad -\hbar^2\frac{\partial^2\psi}{\partial t^2} - \frac{2\hbar^2}{\tau}\frac{\partial\psi}{\partial t} - \frac{\hbar^2}{\tau^2}\psi = m_\circ{}^2 c^4\psi - \hbar^2 c^2\nabla^2\psi \qquad (3.4)$$

This is Klein-Gordon equation of frictional medium. This equation can be obtained also by using the expression for the wave function proposed by M.Dirar and others[3], which takes the form

$$\psi = Ae^{\frac{i}{\hbar}(px-Et)} \qquad (3.5)$$

Thus:

$$i\hbar\frac{\partial}{\partial t}\psi = E\psi$$
$$-\hbar^2\partial_t{}^2\psi = E^2\psi$$

Similarly

$$\frac{\hbar}{i}\nabla\psi = p\psi$$
$$-\hbar^2\nabla^2\psi = p^2\psi \qquad (3.6)$$

But for frictional medium

$$\left(E + \frac{i\hbar}{\tau}\right)^2\psi = c^2p^2\psi + m_\circ{}^2c^4\psi \qquad (3.7)$$
$$E^2\psi + \frac{2i\hbar E}{\tau}\psi - \frac{\hbar^2}{\tau^2}\psi = c^2p^2\psi + m_\circ{}^2c^4\psi \qquad (3.8)$$

Insert (3.6) in (3.8) to get:

$$-\hbar^2\frac{\partial^2\psi}{\partial t^2} - \frac{2\hbar^2}{\tau}\frac{\partial\psi}{\partial t} - \frac{\hbar^2}{\tau^2}\psi = -c^2\hbar^2\nabla^2\psi + m_\circ{}^2c^4\psi \qquad (3.9)$$

This is typical to that obtained by using Lagrangian formalism.

IV. DIRAC EQUATION OF FRICTIONAL MEDIUM:

The wave function formalism [$equations$ (3.5, −3.8)] can also be used to find Dirac equation by using the relation[3]:

$$\left(E + \frac{i\hbar}{\tau}\right)\psi = \alpha.p\psi + \beta m_\circ c^2\psi \qquad (4.1)$$

To get

$$i\hbar\frac{\partial\psi}{\partial t}+\frac{i\hbar}{\tau}\psi=\frac{c\hbar}{i}\alpha.\nabla\psi+\beta m_\circ c^2\psi \qquad (4.2)$$

Thus

$$i\hbar\frac{\partial\psi}{\partial t}=\frac{c\hbar}{i}\alpha.\nabla\psi+\beta m_\circ c^2\psi-\frac{i\hbar}{\tau}\psi \qquad (4.3)$$

This equation can similarly be found by suggesting the Lagrangian

$$L=i\hbar\psi\dot{\psi}^*-\left(\frac{c\hbar}{i}\alpha\nabla\psi^*\right)\psi+\beta m_\circ c^2\psi\psi^*-\frac{i\hbar}{\tau}\psi\psi^* \qquad (4.4)$$

Hence

$$\frac{\partial L}{\partial\psi^*}=\beta m_\circ c^2\psi-\frac{i\hbar}{\tau}\psi$$

$$\frac{\partial L}{\partial\dot{\psi}^*}=i\hbar\psi$$

$$\frac{\partial L}{\partial\partial_x\psi^*}=-\frac{c\hbar\alpha}{i}\psi \qquad (4.5)$$

But

$$\frac{\partial L}{\partial\psi^*}-\partial_t\left[\frac{\partial L}{\partial\dot{\psi}^*}\right]-\partial_x\left[\frac{\partial L}{\partial\partial_x\psi^*}\right]=0 \qquad (4.6)$$

Inserting (4.5) in (4.6) yields

$$\beta m_\circ c^2\psi-\frac{i\hbar}{\tau}\psi-\partial_t[i\hbar\psi]-\partial_x\left[-\frac{c\hbar\alpha}{i}\psi\right]=0$$

$$\therefore \quad i\hbar\frac{\partial\psi}{\partial t}=\frac{c\hbar}{i}\alpha.\nabla\psi+\beta m_\circ c^2\psi-\frac{i\hbar}{\tau}\psi \qquad (4.7)$$

This is Dirac equation of frictional medium, which is typical to that obtained by the wave function approach.

V. SCHRÖDINGER EQUATION SOLUTION FOR CONSTANT POTENTIAL:

For a particle moving in a resistive medium under the effect of constant potential Schrödinger equation (2.8) becomes

$$i\hbar\frac{\partial\psi}{\partial t}=-\frac{\hbar^2}{2m}\nabla^2\psi+V_\circ\psi-\frac{i\hbar\psi}{\tau}=E\psi \qquad (5.1)$$

Now consider the solution

$$\psi=Ae^{\alpha x} \quad \Longrightarrow \frac{\partial\psi}{\partial x}=A\alpha e^{\alpha x} \quad , \quad \frac{\partial^2\psi}{\partial x^2}=A\alpha^2e^{\alpha x} \qquad (5.2)$$

A direct substitution of (5.2) in (5.1) yields

$$-\frac{\hbar^2}{2m}A\alpha^2e^{\alpha x}+V_\circ Ae^{\alpha x}-\frac{i\hbar}{\tau}Ae^{\alpha x}=EAe^{\alpha x}$$

Cancelling exponential term on both sides gives

$$-\frac{\hbar^2}{2m}\alpha^2+V_\circ-\frac{i\hbar}{\tau}=E$$

Thus

$$\frac{\hbar^2}{2m}\alpha^2 = V_\circ - \frac{i\hbar}{\tau} - \frac{\hbar^2 k^2}{2m} \qquad (5.3)$$

If one assumes that

$$E = \frac{\hbar^2 k^2}{2m} \qquad (5.4)$$

Rearranging yields

$$\alpha^2 = \frac{2mV_\circ}{\hbar^2} - \frac{2mi\hbar}{\hbar^2\tau} - k^2$$

Thus

$$\therefore \qquad \alpha = \pm\left[\frac{2m}{\hbar^2}\left(V_\circ - \frac{i\hbar}{\tau}\right) - k^2\right]^{\frac{1}{2}} \qquad (5.5)$$

To find lasing and absorption condition for Schrödinger equation one have to bear in mind that the number of particle are given by

$$n = |\psi|^2 = \psi\bar{\psi} \qquad (5.6)$$

In view of equation (5.2)

$$\psi = Ae^{\alpha z}$$

Thus

$$n = (Ae^{\alpha z})(Ae^{\bar{\alpha} z}) = A^2 e^{(\alpha+\bar{\alpha})z} \qquad (5.7)$$

One can rewrite (5-7) as

$$n = A^2 e^{\pm\gamma z} \qquad (5.8)$$

VI. LASING CONDITION

To see how lasing takes place re call equation (5.5), where

$$\alpha = \left[\frac{2m}{\hbar^2}\left(V_\circ - \frac{i\hbar}{\tau}\right) - k^2\right]^{\frac{1}{2}} \qquad (6.1)$$

For the sake of simplification

$$V_\circ \to 0 \quad and\ k \to o \qquad (6.2)$$

In view of equations (5.5) and (6.1) one gets

$$\alpha = \left(\frac{2m}{\hbar\tau}\right)^{\frac{1}{2}}(-i)^{\frac{1}{2}} \qquad (6.3)$$

But

$$e^{-i\theta} = cos\theta - isin\theta \qquad (6.4)$$

For $(\theta = 90)$ one gets

$$e^{-i90} = cos90 - isin90 = -i$$

Hence

$$\therefore \qquad (-i)^{\frac{1}{2}} = (e^{-90i})^{\frac{1}{2}} = e^{-45i} = cos45 - isin45$$

$$(-i)^{\frac{1}{2}} = \frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}} \qquad (6.5)$$

Thus in view of equations (5.7), (5.8), (6.3) and (6.5)

$$\therefore \quad \alpha+\bar{\alpha}=\left(\frac{2m}{\hbar\tau}\right)^{\frac{1}{2}}\left[\left(\frac{1}{\sqrt{2}}-\frac{i}{\sqrt{2}}\right)+\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}}\right]=2\left(\frac{m}{\hbar\tau}\right)^{\frac{1}{2}} \qquad (6.6)$$

Where

$$\alpha=\left(\frac{2m}{\hbar\tau}\right)^{\frac{1}{2}}.\left(\frac{1}{\sqrt{2}}-\frac{i}{\sqrt{2}}\right)$$

$$\bar{\alpha}=\left(\frac{2m}{\hbar\tau}\right)^{\frac{1}{2}}\left[\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}}\right] \qquad (6.7)$$

In view of equations (5.7), (5.8) and (6.6) by taking the plus sign one gets

$$\therefore \quad n=A^2e^{(\alpha+\bar{\alpha})z}=A^2e^{\gamma z}=A^2e^{\left(\frac{2m}{\hbar\tau}\right)^{\frac{1}{2}}\left(\frac{2}{\sqrt{2}}\right)z}$$

$$n=A^2e^{(\alpha+\bar{\alpha})z}=A^2e^{\gamma z}=A^2e^{2\left(\frac{m}{\hbar\tau}\right)^{\frac{1}{2}}z} \qquad (6.8)$$

VII. ABSORPTION CONDITION

Since the number of particles are given by equation (8.5) to be

$$n=A^2e^{-\gamma z} \qquad (7.1)$$

In view of equation (5.7) it follows that

$$\therefore \quad -\gamma=(\alpha+\bar{\alpha})=-\left(\frac{2m}{\hbar\tau}\right)^{\frac{1}{2}}(-i)^{\frac{1}{2}} \qquad (7.2)$$

$$-\gamma=-\left(\frac{2m}{\hbar\tau}\right)^{\frac{1}{2}}\left[\left(\frac{1}{\sqrt{2}}-\frac{i}{\sqrt{2}}\right)+\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}}\right]=-2\left(\frac{m}{\hbar\tau}\right)^{\frac{1}{2}} \qquad (7.3)$$

Therefore, one gets

$$n=A^2e^{(\alpha+\bar{\alpha})z}=A^2e^{-\gamma z}=A^2e^{-2\left(\frac{m}{\hbar\tau}\right)^{\frac{1}{2}}z} \qquad (7.4)$$

Since n decreases as z increases it follows that absorption takes place.

VIII. KLEIN-GORDON EQUATION SOLUTION FOR RESISTIVE MEDIUM

For Klein-Gordon equation in equation (3.7), one can suggest a solution

$$u=Ae^{\alpha x} \quad \Rightarrow \frac{\partial u}{\partial x}=A\alpha e^{\alpha x} \quad , \quad \frac{\partial^2 u}{\partial x^2}=A\alpha^2e^{\alpha x} \qquad (8.1)$$

Where Klein-Gordon equation takes the form

$$-\hbar^2\frac{\partial^2\psi}{\partial t^2}-\frac{2\hbar^2}{\tau}\frac{\partial\psi}{\partial t}-\frac{\hbar^2}{\tau^2}\psi=m_\circ{}^2c^4\psi-\hbar^2c^2\nabla^2\psi \qquad (8.2)$$

Inserting (8.2) in (8.1) by suggesting

$$\psi=e^{\frac{i}{\hbar}Et}u \qquad (8.3)$$

Yields

$$E^2u+\frac{2i\hbar}{\tau}Eu-\frac{\hbar^2}{\tau^2}u=m_\circ{}^2c^4u-\hbar^2c^2\nabla^2u \qquad (8.4)$$

Now using (8.1) gives

$$E^2Ae^{\alpha x}+\frac{2i\hbar}{\tau}EAe^{\alpha x}-\frac{\hbar^2}{\tau^2}Ae^{\alpha x}=m_\circ{}^2c^4Ae^{\alpha x}-\hbar^2c^2\alpha^2Ae^{\alpha x} \qquad (8.5)$$

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Hence

$$E^2Ae^{\alpha x}+\frac{2i\hbar}{\tau}EAe^{\alpha x}-\frac{\hbar^2}{\tau^2}Ae^{\alpha x}-m_\circ{}^2c^4Ae^{\alpha x}+\hbar^2c^2\alpha^2Ae^{\alpha x}=0 \qquad (8.6)$$

And

$$E^2+\frac{2i\hbar}{\tau}E-\frac{\hbar^2}{\tau^2}-m_\circ{}^2c^4+\hbar^2c^2\alpha^2=0 \qquad (8.7)$$

Therefore

$$\hbar^2c^2\alpha^2=m_\circ{}^2c^4+\frac{\hbar^2}{\tau^2}-\frac{2i\hbar}{\tau}E-E^2$$

As a result α is given by

$$\therefore \qquad \alpha^2=\frac{m_\circ{}^2c^2}{\hbar^2}+\frac{1}{c^2\tau^2}-\frac{2iE}{\hbar c^2\tau}-\frac{E^2}{\hbar^2c^2} \qquad (8.8)$$

Considering

$$E=\frac{\hbar^2k^2}{2m_\circ} \qquad (8.9)$$

Hence

$$\therefore \qquad \alpha^2=\frac{m_\circ{}^2c^2}{\hbar^2}+\frac{1}{c^2\tau^2}-\frac{2i\hbar^2k^2}{2m_\circ\hbar c^2\tau}-\frac{\hbar^2k^4}{4m_\circ{}^2c^2}$$

And

$$\therefore \qquad \alpha=\pm\left[\frac{m_\circ{}^2c^2}{\hbar^2}+\frac{1}{c^2\tau^2}-\frac{i\hbar k^2}{m_\circ c^2\tau}-\frac{\hbar^2k^4}{4m_\circ{}^2c^2}\right]^{\frac{1}{2}} \qquad (8.10)$$

IX. LASING AND ABSORPTION CONDITIONS FOR KLEIN-GORDON EQUATION

In view of equation (8.10), one gets

$$\alpha=\pm\left[\frac{m_\circ{}^2c^2}{\hbar^2}+\frac{1}{c^2\tau^2}-\frac{i\hbar k^2}{m_\circ c^2\tau}-\frac{\hbar^2k^4}{4m_\circ{}^2c^2}\right]^{\frac{1}{2}} \qquad (9.1)$$

For the sake of simplification, let

$$k\to o \qquad (9.2)$$

$$then \qquad \alpha=\gamma=\left[\frac{m_\circ{}^2c^2}{\hbar^2}+\frac{1}{c^2\tau^2}\right]^{\frac{1}{2}} \qquad (9.3)$$

Thus according to equation (6.8) the intensity is given by

$$\therefore \qquad n=A^2e^{\gamma z}=A^2e^{\left(\frac{m_\circ{}^2c^2}{\hbar^2}+\frac{1}{c^2\tau^2}\right)^{\frac{1}{2}}z} \qquad (9.4)$$

Since n increase with z lasing take place.
For absorption

$$\alpha=-\gamma=-\left[\frac{m_\circ{}^2c^2}{\hbar^2}+\frac{1}{c^2\tau^2}\right]^{\frac{1}{2}} \qquad (9.5)$$

And the number of particles is given by

$$\therefore \qquad n=A^2e^{-\gamma z}=A^2e^{-\left(\frac{m_\circ{}^2c^2}{\hbar^2}+\frac{1}{c^2\tau^2}\right)^{\frac{1}{2}}z} \qquad (9.6)$$

As far as no decrease with z absorption takes place.

X. DISCUSSION

The choice of the Lagrargian (2.1) for frictional system within the frame work of Schrödinger equation gives a relation similar to that obtained by M.Dirar and others[3], as shown by equation (2.8). The Lagrangian for Klein-Gordon equation (3.1) gives also a one that accounts for the effect of friction [*equation* (3.4)]. The same equation was found by using conventional wave function approach and Dirar expression for energy in a resistive medium, as equations (3.9,-,3.8) indicates the same hold for Dirac equation where the wave function method , shown by relations (4.1,2,3) gives the same relation far resistive medium by the Lagrargian in equation (4.4). In section (5) Schrödinger equation for constant potential (5.1) was solved by proposing exponential wave function (5.2). Equation (6.8) indicates that lasing can take place, while equation (7.4) predicts absorption. In section (8) Klein-Gordon equation was solved for frictional medium by assuming exponential solution (8.1). The coefficient α is given in equation (8.10) to be strongly dependent on relaxation timeτ. The lasing can exist according to equation (9.4), while absorption is possible also according to equation (9.6).

XI. CONCLUSION

The Klein-Gordon and Dirac Relativistic equation derived by Lagrangian formalism coincides with that found by using conventional approach which is based on string theory. The solution of these equations shows the possibility of lasing and absorption.

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